Math 292 Exam 1 SU26 **Name:** Keitaro Miyai

Graded out of 6 questions. You can complete all 7 but I will only count the 6 highest scores.

Reminder: no calculators, smart watches or any other tech (including smart glasses) are allowed for this exam. You are allowed one index card (2 sided) for notes. Any violations of these conditions result in a zero for the exam.

1) Let $\mathbf{a} = \langle 2, -3, 1 \rangle$, $\mathbf{b} = \langle -4, 1, 2 \rangle$

(a) Compute $\mathbf{a} + \mathbf{b}$ and $3\mathbf{a} - 2\mathbf{b}$

$a+b = \langle 2-4, -3+1, 1+2 \rangle$

$= \langle -2, -2, 3 \rangle$ ✓

$3a-2b = \langle 6+8, -9-2, 3-4 \rangle$

$= \langle 14, -11, -1 \rangle$ ✓

10/10

(b) Find $|\mathbf{a}|$ and a **unit vector** in the direction of $\mathbf{a}$
(Leave answers in exact form with radicals)

$|a| = \sqrt{2^2+(-3)^2+1^2}$

$= \sqrt{4+9+1}$

$= \sqrt{14}$ ✓

Unit vector

$u = \frac{a}{|a|} = \frac{1}{\sqrt{14}} \langle 2, -3, 1 \rangle$

$= \left\langle \frac{2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{1}{\sqrt{14}} \right\rangle$ ✓

(c) Find the angle between $\mathbf{a}$ and $\mathbf{b}$
(Leave answer as $\cos^{-1}(_)$; do NOT approximate)

$a \cdot b = |a||b| \cos\theta$

$\cos\theta = \frac{a \cdot b}{|a||b|}$

$= \frac{-8-3+2}{\sqrt{14}\sqrt{4^2+1^2+2^2}}$

$\cos\theta = \frac{-9}{\sqrt{14 \times 21}}$

$\cos\theta = \frac{-9}{7\sqrt{6}}$

$\therefore \theta = \cos^{-1}\left(\frac{-9}{7\sqrt{6}}\right)$

$16 + 1 + 4 = 21$

$-11+2 = -9$

2.2.

$\frac{-1}{14}$

2)

Let $\mathbf{a} = \langle 2,1,-1\rangle,\ \mathbf{b} = \langle 1,-2,3\rangle$

(a) Compute $\mathbf{a} \times \mathbf{b}$

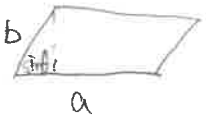
$$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & -1 \\ 1 & -2 & 3 \end{vmatrix}$$

$$= \hat{\imath}(1) - \hat{\jmath}(7) + \hat{k}(-5)$$

$$\mathbf{a}\times\mathbf{b} = \langle 1, -7, -5 \rangle$$

(b) Find the area of the parallelogram determined by **a** and **b**

(Leave answer in exact radical form)



$$A = |\mathbf{a}\times\mathbf{b}|$$

$$= \sqrt{1^2+(-7)^2+(-5)^2}$$

$$= \sqrt{75}$$

$$A = 5\sqrt{3}$$

49 50 75

$1 + 49 + 25 = 75$

(c) Find a unit vector orthogonal to both **a** and **b**

$$\mathbf{u} = \pm\frac{\mathbf{a}\times\mathbf{b}}{|\mathbf{a}\times\mathbf{b}|} = \pm\frac{1}{5\sqrt{3}}\langle 1,-7,-5\rangle$$

$$\mathbf{u} = \left\langle \frac{1}{5\sqrt{3}}, \frac{-7}{5\sqrt{3}}, \frac{-1}{\sqrt{3}} \right\rangle \left\langle \frac{-1}{5\sqrt{3}}, \frac{7}{5\sqrt{3}}, \frac{1}{5\sqrt{3}} \right\rangle$$

3)

(a) Find parametric equations of the line through

$$(2, -1, 4)$$

parallel to the line $x = 1 + 3t,\ y = -2 + t,\ z = 5 - 2t$

The direction vector of the line is

$$d = \langle 3, 1, -2 \rangle$$

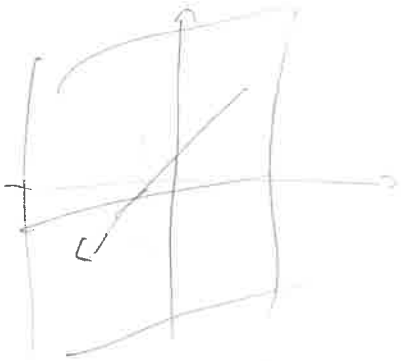
So parametric equations is

$$\begin{cases} x = 2 + 3t \\ y = -1 + t \\ z = 4 - 2t \end{cases}$$

(b) Find the equation of the plane through

$$(1, 0, 2)$$

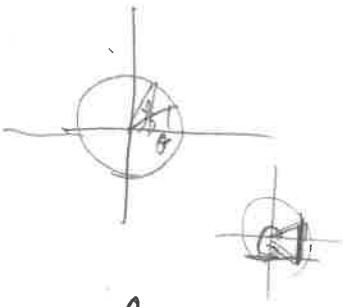
with normal vector $\langle 2, -1, 3 \rangle$



$$2(x-1) - 1(y-0) + 3(z-2) = 0$$

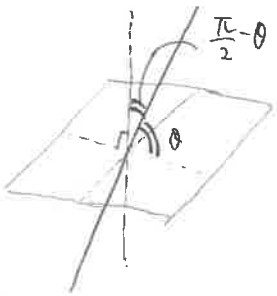
$$2x - y + 3z - 2 - 6 = 0$$

$$2x - y + 3z = 6$$



(c) Find the angle between the line from (a) and the plane from (b)

(Leave answer in inverse trig form; no approximations)



When the angle between the line and the plane is θ, the angle between the line and the normal vector to the plane is $\frac{\pi}{2}-\theta$,

and $\sin\theta = \cos\left(\frac{\pi}{2}-\theta\right)$. --- ① normal vector $n = \langle 2, -1, 3 \rangle$

the direction vector $d = \langle 3, 1, -2 \rangle$.

$$\cos\left(\frac{\pi}{2}-\theta\right) = \frac{n \cdot d}{\|n\|\|d\|}$$

$$= \frac{6 - 1 - 6}{\sqrt{14}\sqrt{14}} = \frac{-1}{14}$$

Using ①,

$$\theta = \sin^{-1}\left(\frac{-1}{14}\right)$$

When $0 \le \theta \le \frac{\pi}{2}$,

$$\theta = \sin^{-1}\left(\frac{1}{14}\right)$$

4)

(a) Find the distance from the point $(3, -2, 1)$

to the plane $x + 2y + 2z = 6$ ··· ①

$\Rightarrow x+2y+2z-6=0$

(Leave your answer in simplified radical form)

$$D = \frac{|3+2(-2)+2(1)-6|}{\sqrt{1^2+2^2+2^2}}$$

$$= \frac{|-5|}{3} = \frac{5}{3}$$ //

$3-4+2-6$

$= -5$

(b) Determine whether the line

$$x = 2 + t, y = -1 + 3t, z = 4 - t \quad \cdots ②$$

intersects the plane from part (a). If so, find the point of intersection.

Plug ② into ①

$(2+t)+2(-1+3t)+2(4-t)=6$

$5t+8=6$

$t=-\frac{2}{5}$

$\therefore \begin{cases} x = \frac{8}{5} \\ y = -\frac{11}{5} \\ z = \frac{22}{5} \end{cases}$ so they intersects each other. //

$\frac{8}{5} - \frac{22}{5} + \frac{44}{5} = \frac{30}{5} = 6$

5)

Given: $\mathbf{r}(t) = \langle t^2, 3t - 1, 2t^3 \rangle$

(a) Find $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$

$r'(t) = \langle 2t, 3, 6t^2 \rangle$

$r''(t) = \langle 2, 0, 12t \rangle$

(b) Find the velocity and acceleration at $t = 1$

$v(1) = r'(1) = \langle 2, 3, 6 \rangle$

$a(1) = r''(1) = \langle 2, 0, 12 \rangle$

(c) Find the equation of the **tangent line** at $t = 1$

$r'(1) = \langle 2, 3, 6 \rangle$

$r(1) = \langle 1, 2, 2 \rangle$

such that

the tangent line at $t=1$ is

$$\begin{cases} x = 1 + 2p \\ y = 2 + 3p \\ z = 2 + 6p \end{cases} \quad (p \in \mathbb{R})$$

6)

A particle moves according to: $\mathbf{r}(t) = \langle 5\cos t,\ 5\sin t,\ t \rangle$

(a) Find velocity and **speed**

(Leave speed in exact radical form)

$v = r'(t) = \langle -5\sin t,\ 5\cos t,\ 1 \rangle$

Let s be speed.

$s = |v| = \sqrt{(-5\sin t)^2 + (5\cos t)^2 + 1^2}$

$s = \sqrt{26}$

$25 + 1 = 26$

(b) Determine whether speed is constant

Whatever t,

$s = \sqrt{(-5\sin t)^2 + (5\cos t)^2 + 1^2}$

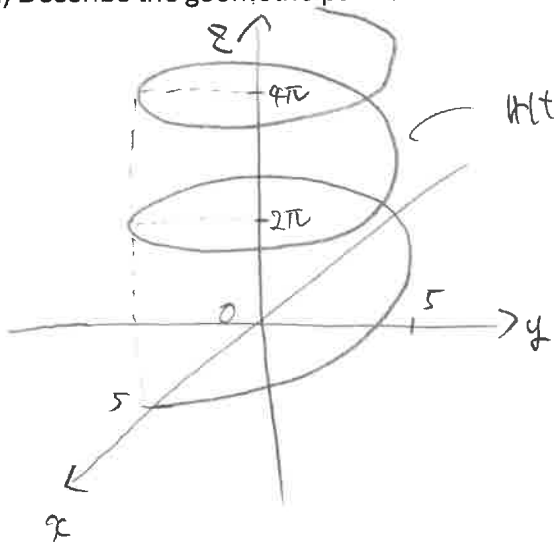
$= \sqrt{26}$ such that speed is constant

(c) Find the acceleration vector

$a = r''(t)$

$a = \langle -5\cos t,\ -5\sin t,\ 0 \rangle$

(d) Describe the geometric path of motion



$r(t) = \langle 5\cos t,\ 5\sin t,\ t \rangle$

7)

A particle has acceleration: $\mathbf{a}(t) = \langle 2, -t, 3t^2 \rangle$

With $\mathbf{v}(0) = \langle 1,0,-1 \rangle$, $\mathbf{r}(0) = \langle 0,2,1 \rangle$

(a) Find velocity $\mathbf{v}(t)$

$v(t) = \langle 2t + C_1, -\frac{1}{2}t^2 + C_2, t^3 + C_3 \rangle$. ($C_1, C_2, C_3$ are const.)

When $t=0$,

$v(0) = \langle C_1, C_2, C_3 \rangle = \langle 1, 0, -1 \rangle$

such that $v(t) = \langle 2t+1, -\frac{1}{2}t^2, t^3 - 1 \rangle$

(b) Find position $\mathbf{r}(t)$

$r(t) = \langle t^2 + t + C_4, -\frac{1}{6}t^3 + C_5, \frac{1}{4}t^4 - t + C_6 \rangle$ (C_4, C_5, C_6 are const.)

When $t=0$,

$r(0) = \langle C_4, C_5, C_6 \rangle = \langle 0, 2, 1 \rangle$

such that $r(t) = \langle t^2 + t, -\frac{1}{6}t^3 + 2, \frac{1}{4}t^4 - t + 1 \rangle$

(c) Find speed at $t = 1$ *(leave in exact form)*

$s(1) = |v(1)|$

$= \sqrt{3^2 + (-\frac{1}{2})^2 + 0^2}$

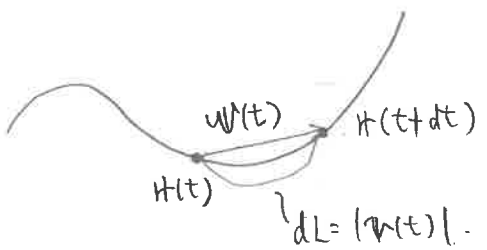
$= \sqrt{\frac{37}{4}} = \frac{\sqrt{37}}{2}$

$9 + \frac{1}{4} + 4 = \frac{}{4}$

$9 + \frac{1}{4} = \frac{37}{4}$

(d) Set up (do NOT evaluate) the integral for total distance on $[0, 1]$





$dL = |r(t+dt) - r(t)|$

$\frac{dL}{dt} = \frac{|r(t+dt) - r(t)|}{dt} = |v(t)|$

$\therefore dL = |v(t)|\,dt$

$L = \int dL = \int |v(t)|\,dt$

$\therefore L = \int_0^1 |v(t)|\,dt = \int_0^1 \sqrt{(t^6 + \frac{1}{4}t^4 - 2t^3 + 4t^2 + 4t + 2)}\,dt$

8) How much time did you spend over the first 2 weeks of the class? What did you spend most of your time on? Homework, reading and taking notes, watching videos or something else?

I spend total 15 hours over the 2 weeks.

I spend most on doing homework and solving problems on the module.

I also spend time on Learn the definition of words because I had never seen some words in problems.