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**Exam II**
**Linear Algebra**
**Math 194**
**7/13/2026 10:05 AM – 12:05 PM**
**Instructor: Meetal Shah**

$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

**Exam Instructions:**
Show work for every question; points will be based on the work shown and answer provided. When using a calculator to do work state what steps are taken on the calculator. Take time to review your work and print clearly. **Print name and DVC ID** at top of **all** pages. This exam is closed book and notes; however, you may use a one-page (two-sided) cheat sheet.

There are eleven questions of 10 points each for a total of 110, but the exam is graded out of 100 points (that is the maximum score is 100).

**Question 1:**
Prove or find a counterexample, if A and B are non-singular n x n matrices then A + B is a non-singular n x n matrix. 正則 正則

Counter example:

If $A = I_n$ and $B = -I_n$, ($\det(A) = 1 \neq 0$, $\det(B) = (-1)^n \neq 0$)

then $A + B = O$ such that $\det(A+B) = 0$, which is singular.

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**Question 2:**

Write **v** = (2,1) as a linear combination of **u** and **w**, if possible, where **u** = (1,2) and **w** = (1, -1).

When $v = su + tw \quad (s, t \in \mathbb{R})$,

$$\begin{bmatrix} 2 \\ 1 \end{bmatrix} = s\begin{bmatrix} 1 \\ 2 \end{bmatrix} + t\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\Rightarrow \begin{cases} 2 = s + t \\ 1 = 2s - t \end{cases}$$

$$\Rightarrow \begin{cases} s = 1 \\ t = 1 \end{cases}$$

Since then, $v = u + w$ //

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**Question 3:**

Determine whether the set of all 2x2 diagonal matrices together with the standard operations is a vector space. If not, identify at least 1 of the 10 vector space properties that fails.

Let $W$ be the set of 2x2 diagonal matrices,

$$W = \begin{bmatrix} n_1 & 0 \\ 0 & n_2 \end{bmatrix} \quad (n_1, n_2 \in \mathbb{R}).$$

When $n_1 = n_2 = 0$, $\begin{bmatrix} n_1 & 0 \\ 0 & n_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 \in W$ ... ①

When $\begin{bmatrix} n_1 & 0 \\ 0 & n_2 \end{bmatrix} + \begin{bmatrix} n_1' & 0 \\ 0 & n_2' \end{bmatrix} = \begin{bmatrix} n_1+n_1' & 0 \\ 0 & n_2+n_2' \end{bmatrix} \in W$ ... ②,

When $c \in \mathbb{R}$,

$$c\begin{bmatrix} n_1 & 0 \\ 0 & n_2 \end{bmatrix} = \begin{bmatrix} cn_1 & 0 \\ 0 & cn_2 \end{bmatrix} \in W \quad \cdots ③,$$

so the set of all 2x2 diagonal matrices is a vector space. ☒

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**Question 4:**

Let V = C(-∞, ∞) be the set of continuous functions on the real line. Then V is a vector space with the usual operations of function addition and scalar multiplication of functions. Determine if the set of all positive functions: $f(x) > 0$ is a subspace of C(-∞, ∞).

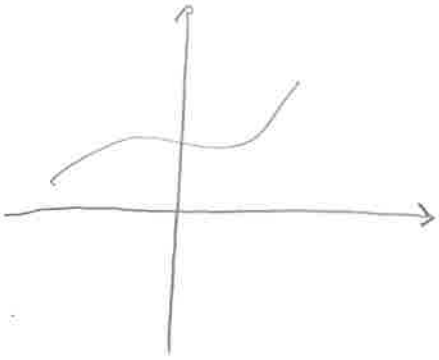
$f(x) > 0$ does not include

$f(x) = 0$, which means $f(x) > 0$ doesn't satisfy

the Theorem "Subspace includes $0 \in W$."

∴

so, $f(x) > 0$ is not a subspace. ⊠



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**Question 5:**

**Prove that any set of vectors from a vector space containing the zero vector is linearly dependent.**

Let $V=\{v_n (1 \le n \le t)\}$ be a set of vectors containing a zero vector.

If $V$ is linearly independent, then

$c_1 v_1 + c_2 v_2 + \cdots\cdots + c_t v_t = 0$ has only one solution

, which is $c_1 = c_2 = \cdots\cdots = c_t = 0$.

However, if $V$ contains $\vec{0}$, for example, $v_n = 0$,

$c_n$ can be any real number, which means

$c_1 v_1 + c_2 v_2 + \cdots\cdots + c_t v_t = 0$ has infinite solutions.

Such that, $V$ contains $\vec{0}$ is linearly dependent. ☒

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**Question 6:**

Explain why S is not a basis for $P_2$.

$S = \{1 - 2x + x^2, 3 - 6x + 3x^2, -2 + 4x - 2x^2\}$

$3 - 6x + 3x^2 = 3(1 - 2x + x^2)$, it means S is linearly dependent,

so S is not a basis for $P_2$. ■

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**Question 7:**
Find the nullspace of the matrix $A = [1 \quad 2 \quad 3]$.

$Ax = 0$

$$[1\ 2\ 3]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

So

$$x_1 + 2x_2 + 3x_3 = 0$$

Let $x_2$, $x_3$ be $x_2 = s$, $x_3 = t$.

$x_1 = -2s - 3t$

$x_2 = s$

$x_3 = t$ such that

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = s\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}, \text{ so,}$$

the null space is $\begin{bmatrix} -2 & -3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$. ☒

$$1\ 2\ 3 \begin{bmatrix} -2 & -3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$1\times3 \quad 3\times2$

$= [0 \quad 0]$

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**Question 8:**

Find the coordinate matrix of **x** in $\mathbb{R}^n$ relative to the basis B'.

$B' = \{(-5, 6), (3,-2)\}$, $\mathbf{x} = (-17, 22)$

Let $[x]_{B'} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ be the coordinate matrix of $X$ in $\mathbb{R}^n$ relative to the basis $B'$.

$$B'[x]_{B'} = X$$

$$\begin{bmatrix} -5 & 3 \\ 6 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -17 \\ 22 \end{bmatrix}$$

so, $\begin{cases} -5x_1 + 3x_2 = -17 \\ 6x_1 - 2x_2 = 22 \end{cases}$

$\Leftrightarrow \begin{cases} x_1 = 4 \\ x_2 = 1 \end{cases}$

such that $[x]_{B'} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ ☒

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**Question 9:**

$$\begin{bmatrix} 1 & -2 & 7 & 9 \\ 3 & -4 & 5 & 5 \\ 3 & 6 & 1 & -1 \\ 4 & 5 & 3 & 2 \end{bmatrix}$$

Find the determinant of the matrix above.

$$\begin{bmatrix} 1 & -2 & 7 & 9 \\ 3 & -4 & 5 & 5 \\ 3 & 6 & 1 & -1 \\ 4 & 5 & 3 & 2 \end{bmatrix}$$

$\downarrow R_2 - 3R_1 \to R_2,\ R_3 - 3R_1 \to R_3,$
$R_4 - 4R_1 \to R_4$

$$\begin{bmatrix} 1 & -2 & 7 & 9 \\ 0 & 2 & -16 & -22 \\ 0 & 12 & -20 & -28 \\ 0 & 13 & -25 & -34 \end{bmatrix}$$

$\downarrow R_2 \to \frac{1}{2}R_2$

$$\begin{bmatrix} 1 & -2 & 7 & 9 \\ 0 & 1 & -8 & -11 \\ 0 & 12 & -20 & -28 \\ 0 & 13 & -25 & -34 \end{bmatrix}$$

$\downarrow R_1 + 2R_2 \to R_1,\ R_3 - 12R_2 \to R_3,$
$R_4 - 13R_2 \to R_4$

$$\begin{bmatrix} 1 & 0 & -9 & -13 \\ 0 & 1 & -8 & -11 \\ 0 & 0 & 76 & 104 \\ 0 & 0 & 79 & 109 \end{bmatrix}$$

$\downarrow R_4 - R_3 \to R_4$

$$\begin{bmatrix} 1 & 0 & -9 & -13 \\ 0 & 1 & -8 & -11 \\ 0 & 0 & 76 & 104 \\ 0 & 0 & 3 & 5 \end{bmatrix}$$

$\downarrow R_3 - 25R_4 \to R_3$

$$\begin{bmatrix} 1 & 0 & -9 & -13 \\ 0 & 1 & -8 & -11 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 3 & 5 \end{bmatrix}$$

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(I did it on the next page.

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$4-30=-26$

$$\begin{vmatrix} 1 & -2 & 7 & 9 \\ 3 & -4 & 5 & 5 \\ 3 & 6 & 1 & -1 \\ 4 & 5 & 3 & 2 \end{vmatrix}$$

$-12$
$25$
$37$

$$= 1\times\begin{vmatrix} -4 & 5 & 5 \\ 6 & 1 & -1 \\ 5 & 3 & 2 \end{vmatrix} \quad (= 1\times\{-4(5)-5(17)+5(13)\} = -20-85+65 = -40.$$

$129$
$105$
$24$

$$-3\begin{vmatrix} -2 & 7 & 9 \\ 6 & 1 & -1 \\ 5 & 3 & 2 \end{vmatrix} \quad (= -3\times\{-2(5)-7(17)+9(13)\} = -3\times(-10-119+107)$$
$$= -3\times(-22) = 66$$

$333$
$241$
$92$

$$+3\begin{vmatrix} -2 & 7 & 9 \\ -4 & 5 & 5 \\ 5 & 3 & 2 \end{vmatrix} \quad (= 3\{-2(-5)-7(-33)+9(-37)\} = 3(10+231-333)$$
$$= 3\times(-92) = -276$$

$306$
$69$
$237$

$$-4\begin{vmatrix} -2 & 7 & 9 \\ -4 & 5 & 5 \\ 6 & 1 & -1 \end{vmatrix} \quad (= -4\{-2(-10)-7(-26)+9(-34)\} = -4(20+182-306)$$
$$= -4\times(-104) = 416$$

$$= -40+66-276+416$$

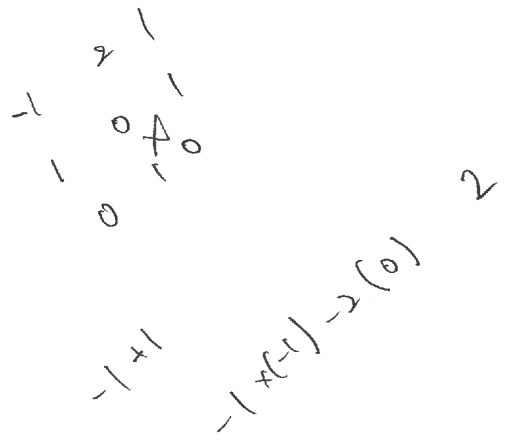
$$= 166$$

$948$
$\times 26$
$1224$
$66$
$1290$
$-\ 34$
$1256$

$306$
$202$
$104$

$482$
$316$
$166$

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**Question 10:**
Let

$$A = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Find |AB|.

$$AB = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 & 3 \\ -1 & 0 & 3 \\ 0 & 2 & 0 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} 1 & 4 & 3 \\ -1 & 0 & 3 \\ 0 & 2 & 0 \end{vmatrix}$$

$$= 2 \times (-1)^5 \times \begin{vmatrix} 1 & 3 \\ -1 & 3 \end{vmatrix}$$

$$= -2 \times 6$$

$$= -12$$

$-1 \times 1$

$-1$

$-1 \times (-1) - (1 \times -1) = 2$

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$$\begin{vmatrix} i & j & k \\ -2 & 1 & 6 \\ 1 & 1 & 2 \end{vmatrix}$$

$= \langle -4, 10, -3 \rangle$

$-4x - 10y - 3z = -7$

$-4 + 20 - 3 = 13 \quad -27$

$-4x \qquad 4x + 10y + 3z = 27$

$4x + 3b - 10z = 27$

$4 + 6 - 10 = 0$

$4x - 10y + 3z = 27$

$4 + 20 + 3 = 27$

$-4 + 10 + 21 = 27$

$8 + 10 + 9 = 27$

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**Question 11:**
Find the equation of the plane passing through the points.

(1, -2, 1), (-1, -1, 7), (2, -1, 3)

When the equation of the plane is $ax+by+cz=d$, $(a,b,c,d \in \mathbb{R})$

then
$$\begin{cases} a - 2b + c = d \\ -a - b + 7c = d \\ 2a - b + 3c = d \end{cases}$$

such that
$$\begin{cases} 3a = \frac{4}{9}d \\ -3b = \frac{10}{9}d \\ 9c = d \end{cases}$$

By using augmented matrix,

$$\begin{bmatrix} 1 & -2 & 1 & 1 \\ -1 & -1 & 7 & 1 \\ 2 & -1 & 3 & 1 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -2 & 1 & 1 \\ 0 & -3 & 8 & 2 \\ 0 & 3 & 1 & -1 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 3 & -6 & 3 & 3 \\ 0 & -3 & 8 & 2 \\ 0 & 3 & 1 & -1 \end{bmatrix}$$

$$\downarrow \quad -\frac{1}{9} + \frac{B}{9}$$

$$\begin{bmatrix} 3 & 0 & -13 & -1 \\ 0 & -3 & 8 & 2 \\ 0 & 0 & 9 & 1 \end{bmatrix} \quad 2 - \frac{8}{9} = \frac{10}{9}$$

$$\downarrow$$

$$\begin{bmatrix} 3 & 0 & 0 & \frac{4}{9} \\ 0 & -3 & 0 & \frac{10}{9} \\ 0 & 0 & 9 & 1 \end{bmatrix}$$

When $d = 27$,

$$\begin{cases} a = 4 \\ b = -10 \\ c = 3 \\ d = 27 \end{cases}$$

so $4x - 10y + 3z = 27$ //

$$\begin{cases} 4 + 20 + 3 = 27 \\ -4 + 10 + 21 = 27 \\ 8 + 10 + 9 = 27 \end{cases}$$

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