Keitaro Miyai DVC ID: 2187549

Exam I
Linear Algebra
Math 194
6/25/2026 10:05 AM – 12:05 PM
Instructor: Meetal Shah

Exam Instructions:
Show work for every question; points will be based on the work shown and answer provided. When using a calculator to do work state what steps are taken on the calculator. Take time to review your work and print clearly. **Print name and DVC ID** at top of **all** pages. This exam is closed book and notes; however, you may use a one-page (two-sided) cheat sheet.

There are eleven questions of 10 points each for a total of 110, but the exam is graded out of 100 points (that is the maximum score is 100).

Question 1:
Solve

$$x + y + z = 1$$

Let y, z be $y = t_1$, $z = t_2$ $(t_1 \in \mathbb{R}, t_2 \in \mathbb{R})$

such that

$$x + t_1 + t_2 = 1$$

$$\begin{cases} x = 1 - t_1 - t_2 \\ y = t_1 \\ z = t_2 \end{cases}$$

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Question 2:

Solve the following system of equations by writing its corresponding augmented matrix and using Gauss-Jordan elimination or Gaussian elimination and back substitution.

$$x - 3z = -2$$
$$3x + y - 2z = 5$$
$$2x + 2y + z = 4$$

If 2K/7

Augmented Matrix

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 3 & 1 & -2 & 5 \\ 2 & 2 & 1 & 4 \end{bmatrix}$$

$-3R_1 \to R_2$

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 2 & 2 & 1 & 4 \end{bmatrix}$$

$-2R_1 \to R_3$

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 2 & 7 & 8 \end{bmatrix}$$

$-2R_2 \to R_3$

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 1 & 0 & -3 \end{bmatrix}$$

$-1R_3 \to R_2$

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 0 & 0 & 7 & 14 \\ 0 & 1 & 0 & -3 \end{bmatrix}$$

$\frac{1}{7}R_2 \to R_2$, $R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$R_1 + 3R_3 \to R_1$

$$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

such that

$$\begin{cases} x = 4 \\ y = -3 \\ z = 2 \end{cases}$$

Back sub^{n}

$4 - 3 \times 2 = -2$

$3 \times 4 - 3 - 2 \times 2 = 5$

$2 \times 4 + 2(-3) + 2 = 4$

$8 - 6 + 2 = 4$

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$$\begin{matrix} 1 & 0 & -3 & -2 \\ 3 & 1 & -2 & 5 \\ 2 & 2 & 1 & 4 \end{matrix}$$

$$\begin{matrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 2 & 7 & 8 \end{matrix}$$

$$\begin{matrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 1 & 0 & -3 \end{matrix}$$

$$\begin{matrix} 1 & 0 & -3 & -2 \\ 0 & 0 & 7 & 14 \\ 0 & 1 & 0 & -3 \end{matrix}$$

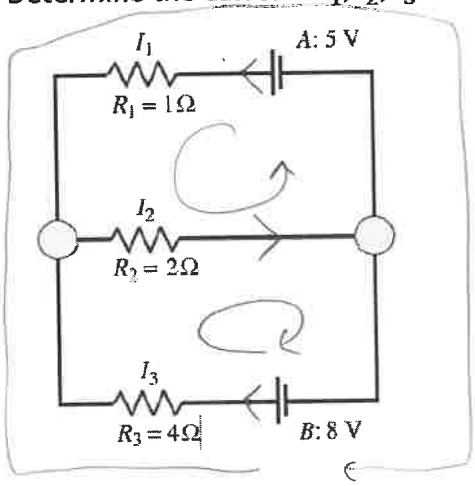
$$\begin{matrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{matrix}$$

$$\begin{matrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{matrix}$$

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Question 3:

Determine the currents I_1, I_2, I_3 for the electrical network shown below.



Based on the picture,

$$\begin{cases} 5 - 1\times I_1 - 2\times I_2 = 0 \\ 8 - 4\times I_3 - 2\times I_2 = 0 \\ I_1 - I_2 + I_3 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} I_1 + 2I_2 = 5 \\ 2I_2 + 4I_3 = 8 \\ I_1 - I_2 + I_3 = 0 \end{cases}$$

Making Augmented Matrix,

$$\begin{bmatrix} 1 & 2 & 0 & 5 \\ 0 & 2 & 4 & 8 \\ 1 & -1 & 1 & 0 \end{bmatrix}$$

$R_3 - R_1 \to R_3$

$$\begin{bmatrix} 1 & 2 & 0 & 5 \\ 0 & 2 & 4 & 8 \\ 0 & -3 & 1 & -5 \end{bmatrix}$$

$R_2 - 4R_3 \to R_2$

$$\begin{bmatrix} 1 & 2 & 0 & 5 \\ 0 & 14 & 0 & 28 \\ 0 & -3 & 1 & -5 \end{bmatrix}$$

$\frac{1}{14}R_2 \to R_2$

$$\begin{bmatrix} 1 & 2 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & -3 & 1 & -5 \end{bmatrix}$$

$R_1 - 2R_2 \to R_1$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & -3 & 1 & -5 \end{bmatrix}$$

$3R_2 + R_3 \to R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

such that

$$\begin{cases} I_1 = 1 \ (A) \\ I_2 = 2 \ (A) \\ I_3 = 1 \ (A) \end{cases}$$

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$$\begin{array}{cccc} 1 & 2 & 0 & 5 \\ 0 & 2 & 4 & 8 \\ 0 & 2 & 0 & 0 \end{array}$$

$$\begin{array}{cccc} 1 & 0 & 0 & 5 \\ 0 & 0 & 4 & 8 \\ 0 & 1 & 0 & 0 \end{array}$$

$I_1 = 5$

$I_2 = 0$

$I_3 = 2$

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Question 4:

Find, if possible. If impossible give reason.

a) **(5 points)** AB

b) **(5 points)** BA

$$A = \begin{bmatrix} 3 & 2 & 1 \\ -3 & 0 & 4 \\ 4 & -2 & -4 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 1 & -2 \end{bmatrix}$$

3×3 $\qquad$ 3×2

a) AB

$$= \begin{bmatrix} 3 & 2 & 1 \\ -3 & 0 & 4 \\ 4 & -2 & -4 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 3\times1+2\times2+1\times1 & 3\times2+2\times(-1)+1\times(-2) \\ -3\times1+0\times2+4\times1 & -3\times2+0\times(-1)+4\times(-2) \\ 4\times1+(-2)\times2+(-4)\times1 & 4\times2+(-2)\times(-1)+(-4)\times(-2) \end{bmatrix}$$

$$\begin{bmatrix} 3+4+1 & 6-2-2 \\ -3+4 & -6-8 \\ 4-4-4 & 8+2+8 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 2 \\ 1 & -14 \\ -4 & 18 \end{bmatrix}$$

b) That is impossible

because B's column is 2, and

A's row is 3, so

B's column and A's row is not equal

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Question 5:

Show that $AC = BC$, even thou $A \neq B$.

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$$

AC

$$= \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0\times 2 + 1\times 2 & 0\times 3 + 1\times 3 \\ 0\times 2 + 1\times 2 & 0\times 3 + 1\times 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$$

BC

$$= \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times 2 + 0\times 2 & 1\times 3 + 0\times 3 \\ 1\times 2 + 0\times 2 & 1\times 3 + 0\times 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$$

such that

$AC = BC = \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$ ☒

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Question 6:

a) (2 points) Write the matrix equation corresponding to the following system of equations. Clearly identify all matrices in the equation.

b) (5 points) Find the corresponding matrix inverse if it exists.

c) (3 points) Use the inverse if it exists to solve.

$$x + 2y + z = 1$$
$$x - 2y - z = 3$$
$$x - 2y + z = -3$$

$$\begin{cases} x+2y+z=1 \\ x-2y-z=3 \\ x-2y+z=-3 \end{cases}$$

$$\Rightarrow \begin{pmatrix} 1 & 2 & 1 \\ 1 & -2 & -1 \\ 1 & -2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -3 \end{pmatrix} \quad (a)$$

Let A, X, B be

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & -2 & -1 \\ 1 & -2 & 1 \end{pmatrix}$$

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$B = \begin{pmatrix} 1 \\ 3 \\ -3 \end{pmatrix}$$

The equation is

$$AX = B$$
$$\Rightarrow A^{-1}AX = A^{-1}B$$
$$\Rightarrow X = A^{-1}B \quad \cdots ①$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 1 & -2 & -1 & 0 & 1 & 0 \\ 1 & -2 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_2 - R_1 \to R_2$, $R_3 - R_1 \to R_3$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -4 & -2 & -1 & 1 & 0 \\ 0 & -4 & 0 & -1 & 0 & 1 \end{array}\right]$$

$R_2 - R_3 \to R_2$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & -2 & 0 & 1 & -1 \\ 0 & -4 & 0 & -1 & 0 & 1 \end{array}\right]$$

$\frac{1}{-2}R_2 \to R_2$, $\frac{1}{-4}R_3 \to R_3$, $R_2 \leftrightarrow R_3$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 & \frac{1}{4} & 0 & -\frac{1}{4} \\ 0 & 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} \end{array}\right]$$

So $A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & -\frac{1}{4} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$ (b)

Using ①,

$$X = A^{-1}B = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & -\frac{1}{4} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{pmatrix} 1 \\ 3 \\ -3 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} + \frac{3}{2} \\ \frac{1}{4} + \frac{3}{4} \\ -\frac{3}{2} - \frac{3}{2} \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

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$$\begin{bmatrix} 1 & 2 & 0 & 5 \\ 0 & 2 & 4 & 8 \\ 1 & -1 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 & 5 \\ 0 & 2 & 4 & 8 \\ 0 & -3 & 1 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 & 5 \\ 0 & 14 & 0 & 28 \\ 0 & -3 & 1 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & -3 & 1 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$3\times 3 \times 3\times 2$.

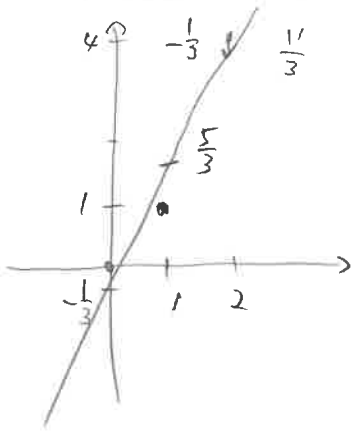
$$\begin{bmatrix} 3 & 2 & 1 \\ -3 & 0 & 4 \\ 4 & -2 & -4 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 1 & -2 \end{bmatrix}$$

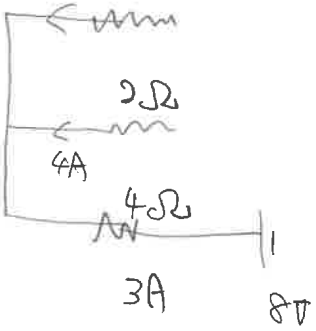
$$= \begin{bmatrix} 8 & 2 \\ 1 & -14 \\ -4 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & -2 & -1 \\ 1 & -2 & 1 \end{bmatrix} \quad \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & -\frac{1}{4} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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Question 7:

Find a sequence of elementary matrices that can be used to row-reduce the following matrix to row-echelon form.

$$\begin{bmatrix} 0 & 1 & 7 \\ 5 & 10 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{5} \end{bmatrix} \begin{bmatrix} 0 & 1 & 7 \\ 5 & 10 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 7 \\ 1 & 2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 7 \\ 1 & 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 7 \end{bmatrix}$$

So, the sequence of elemtary matrices are

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{5} \end{bmatrix}$$

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Question 8:
Find the least squares regression line for the points (0,0), (1,1), (2,4).

Let $X = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{pmatrix}$, $Y = \begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix}$,

and A is the matrix that I have to get,

$A = (X^T X)^{-1} X^T Y$

$X^T X = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{pmatrix}$

$= \begin{bmatrix} 3 & 3 \\ 3 & 5 \end{bmatrix}$

$\begin{bmatrix} 3 & 3 \\ 3 & 5 \end{bmatrix}^{-1} = \frac{1}{3\times5 - 3\times3} \begin{bmatrix} 5 & -3 \\ -3 & 3 \end{bmatrix}$

$= \frac{1}{6} \begin{bmatrix} 5 & -3 \\ -3 & 3 \end{bmatrix}$

$\frac{1}{6} \begin{bmatrix} 5 & -3 \\ -3 & 3 \end{bmatrix} \times \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix}$

$\frac{1}{6} \begin{bmatrix} 5 & 2 & -1 \\ -3 & 0 & 3 \end{bmatrix}$

$\frac{1}{6} \begin{bmatrix} 5 & 2 & -1 \\ -3 & 0 & 3 \end{bmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix}$

$\frac{1}{6} \begin{pmatrix} 2 - 4 \\ 12 \end{pmatrix}$

$= \frac{1}{6} \begin{pmatrix} -2 \\ 12 \end{pmatrix}$

$= \begin{pmatrix} -\frac{1}{3} \\ 2 \end{pmatrix}$

so the line is

$y = 2x - \frac{1}{3}$

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Question 9:

a) **(4 points)** Write an example of a system of homogeneous linear equations.

b) **(3 points)** How many solutions (if any) does your homogeneous system of linear equations have?

c) **(3 points)** Write an example of an inconsistent system of linear equations and state why it is inconsistent.

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a)

$$x + y = 0 \quad \cdots ①$$
$$x - y = 0 \quad \cdots ②$$

//

b) ① + ②

$$2x = 0$$
$$x = 0, \quad y = 0$$

so that has only one solutions.

//

c)

$$x - y = 0 \quad \cdots ①$$
$$2x - 2y = 0 \quad \cdots ②$$

//

① × 2 = ②, so

there is only 1 equation for solve that, and there are two variables, such that there are infinite solutions that mean inconsistent.

$$\begin{cases} x = t & (t \in \mathbb{R}) \\ y = t \end{cases}$$

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Question 10:

Prove the associative property of scalar multiplication for matrices, that is:

$$(cd)A = c(dA)$$

Where c, d are scalars and A is a matrix.

When A is $m \times n$ matrix and,

the box of i Row j column of A is, cij $(1 \le i \le m, 1 \le j \le n)$

think about $(cd)A$,

the box of i Row j column of $(cd)A$ matrix

is $(cd)cij$. ···①

Think about dA,

the box of i Row j column of dA matrix

is $dcij$ and then, think about $c(dA)$ matrix,

the box of i Row j column of $c(dA)$ matrix

is $c(dcij)$ ···②

Because of ① and ②, $(cd)cij = c(dcij)$

so $(cd)A = c(dA)$ //

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Question 11:

If A is an invertible matrix, and c is a non-zero scalar then prove the property below.

$$(cA)^{-1} = \frac{1}{c}A^{-1}$$

$(cA) \times \frac{1}{c} A^{-1}$

$= c \times \frac{1}{c} \times A \times A^{-1}$

$= I$

such that

$\frac{1}{c} A^{-1}$ is the inverse of (cA)

because (cA) times $\frac{1}{c} A^{-1}$ is I.

Since that,

$(cA)^{-1} = \frac{1}{c} A^{-1}$ //