Chapter 1

Linear Equations

Systems of Linear Equations

Solution of a Linear Equation(s)

Solution set of a Linear Equation(s)

Parametric Representation

Number of Solutions to a System of Linear Equations

Back Substitution

Row-Echelon Form

Gaussian Elimination

Inconsistent System of Linear Equations

Systems of Linear Equations with Infinitely Many Solutions

Matrices

Augmented Matrix

Size of a Matrix

Elementary Row Operations

Reduced Row Echelon Form

Gaussian Elimination with Back Substitution

Gauss-Jordan Elimination

Homogeneous System of Linear Equations

Number of Solutions to Homogeneous Systems of Linear Equations

Polynomial Curve Fitting

Network Analysis

Analysis of an Electrical Network

Chapter 2

Equality of Matrices

Matrix Addition and Subtraction

Matrix Scalar multiplication

Matrix Multiplication

Systems of Linear Equations as Matrix equations

Partition of Matrices

Linear Combination of Vectors

Algebra of Matrices

Properties of Matrix Addition and Scalar Multiplication

Zero Matrix

Properties of Zero Matrices

Solving Matrix Equations

Properties of Matrix Multiplication

Identity Matrix

Properties of the Identity Matrix

Number of Solutions of a Linear System

Transpose of a Matrix

Properties of Transposes

Inverse Matrices

Uniqueness of the Inverse Matrix

Finding an Inverse Matrix by Gauss-Jordan Elimination

Singular Matrices

Properties of Inverse Matrices

Inverse of a Product

Cancellation Properties

Systems of Equations with Unique Solutions

Elementary Matrices

Elementary Row Operations

Representing Elementary Row Operations

Using Elementary Matrices

Row Equivalence

Properties of Elementary Matrices

Properties of Invertible Matrices

Writing a Matrix as a Product of Elementary Matrices

Equivalent Conditions

LU-factorization

Finding an LU-Factorization

Solving a System of Linear Equations with LU-factorization

Cryptography

Encoding/Decoding Messages

Least Squares Regression Analysis

Least Squares Regression Line

Matrix Form for Linear Regression

Finding the Least Squares Regression Line

Chapter 3

Determinants

Minors

Cofactors

Determinants by expansion by co-factors

3x3 Determinants (Arrow way)

Determinant of Triangular Matrices

Elementary Row Operations and Determinants

Elementary Column Operations and Determinants

Conditions that give a Zero Determinant

Compute Determinants using Row and Column Operations

Properties of Determinants

Determinant of a Product

Determinant of a scalar multiple of a Matrix

Determinants and Invertibility

Determinant of an Inverse

Equivalent Conditions for a non-singular Matrix

Determinant of a Transpose

Classical Adjoint

Inverse defined by classical adjoint

Cramer's Rule

Cramer's Rule for nxn system

Area of a Triangle with determinants

Collinearity by determinants

Volume of a Tetrahedron by determinants

Coplanarity by determinants

Chapter 4

Vectors in $\mathbb{R}^n$

n-tuple

Vectors

Sum and Difference of Vectors

Scalar multiplication of vectors

Properties of Vector Addition and Scalar Multiplication

Equality of Vectors

Zero vector

Additive Inverse

Properties of Additive Inverse and Additive Identity

Linear Combination of vectors

Vector Spaces

Examples of Vector Spaces

Properties of Scalar Multiplication

Subspaces

Test for a Subspace

Trivial and Proper Subspaces

Nested Subspaces

Intersection of Subspaces

Subspaces of $\mathbb{R}^2$, $\mathbb{R}^3$

Linearly Dependent set of Vectors

Linearly Independent set of Vectors

Spanning Sets

Standard Spanning Sets

Span of a set S

Test for Linear Independence

Properties of Linearly Dependent Sets

Corollary of Theorem on Linearly Dependent sets

Basis

Dimension

Standard Bases

Uniqueness of Basis Representation

Bases and Linear Dependance

Property of number of vectors in a basis

Basis Tests in n-dimensional space

Rowspace of a matrix

Column space of a matrix

Theorem on Rowspace of row-equivalent matrices

Basis for the rowspace of a matrix

2 Methods for finding basis for column space of a matrix

Theorem on Dimension of Row and Column spaces of a matrix

Rank of a matrix

Theorem on Solutions of a Homogenous System

Nullspace of a matrix

Nullity of a matrix

Theorem on the Dimension of the solution space (Rank + Nullity Theorem)

Theorem on Solutions of a Non-Homogenous System

Theorem on Consistency of System of Non-Homogenous System

Summary of Equivalent Conditions for Square Matrices

Coordinate representation of a vector relative to a basis

Transition Matrix

Inverse of a Transition Matrix

Transition Matrix from B to B’

Chapter 5

Lenth, norm or magnitude of a vector

Unit vector

Parallel vectors

Same and opposite direction of vectors

Length of a scalar multiple of a vector

Unit vector in the direction of v

Distance between vectors

Properties of distance

Dot Product of Vectors

Properties of Dot product

Euclidean n-space

Cauchy-Schwarz Inequality

Angle between Vectors

Orthogonal vectors

Triangle Inequality

Pythagorean Theorem

Inner Product

Inner Product Space

Examples of Inner Product Spaces

Properties of Inner Products

Lenth, norm or magnitude of a vector (general)

Distance between vectors (general)

Angle between Vectors (general)

Orthogonal vectors (general)

Unit vector in the direction of v (general)

Properties of Length and Distance (general)

Cauchy-Schwarz Inequality (general)

Triangle Inequality (general)

Pythagorean Theorem (general)

Orthogonal Projections

Theorem on Orthogonal Projections and Distance

Orthogonal Set

Orthonormal Set

Orthogonal Basis

Orthonormal Basis

Theorem on Linear Independence of Orthogonal Sets

Corollary to Theorem on Linear Independence of Orthogonal Sets

Coordinates relative to an orthonormal basis

Gram-Schmidt Orthonormalization process

Gram-Schmidt Orthonormalization process for subspaces

Alternate Gram-Schmidt Orthonormalization process

Least Squares Problem

Orthogonal Subspaces

Orthogonal Complement

Direct Sum

Theorem on Subspaces

Projection onto a subspace

Orthogonal Projection and Distance

Fundamental subspaces

Theorem on Fundamental Subspaces

Normal equations to least squares problem

Mathematical modelling with least squares problem

Chapter 6

Maps

Domain

Codomain

Image

Range

Preimage

Linear Transformations

Zero Transformation

Identity Transformation

Properties of Linear Transformations

Theorem on Linear Transformations Given by a Matrix

Rotations

Projections

Kernel of a Linear Transformation

Theorem on the Kernel of a Linear Transformation

Corollary to Theorem on the Kernel of a Linear Transformation

Theorem on Range of a Linear Transformation

Corollary to Theorem on Range of a Linear Transformation

Nullity of a Linear Transformation

Rank of a Linear Transformation

Theorem on Sum of Rank and Nullity

1-1 Transformations

Onto Transformations

Theorem on 1-1 Transformations

Theorem on Onto and 1-1 Linear Transformations

Isomorphism

Theorem on Isomorphism and Dimension

Examples of Isomorphism

Standard Matrix for a Linear Transformation

Composition of Linear Transformations

Theorem on Composition of Linear Transformations

Inverse Transformation

Invertible Transformations

Theorem on Existence of Inverse Linear Transformations

Transformation Matrix for a non-standard basis

Matrix of a Linear Transformation relative to the bases B and B’

Finding a Linear Transformation relative to a basis

Similar Matrices

Properties of Similar Matrices

Diagonal Matrices

Powers of Diagonal Matrices

Reflections represented as Linear Transformations

Expansions represented as Linear Transformations

Contractions represented as Linear Transformations

Shear Transformations

Rotation transformations

Right hand rule for orientation

Topics in **BOLD** WILL be on the Exam, other topics may or may not be included.

A cheat sheet of one page (both sides) handwritten is allowed, the exam is closed book/notes.

The Exam will consist of 11 questions of 10 points each for a total of 110 points (10 points extra credit), however scores are capped at a maximum score of 100.

Exam Day will consist of a 2-hour Exam in-person