Name: _____ **Class:** _____ **Date:** _____

final review

Numeric Response

1. Find the directional derivative of $f(x,y) = 2\sqrt{x} - y^2$ at the point (1, 3) in the direction toward the point (3, 1).

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2. Find the maximum rate of change of $f(x,y) = x^2y + \sqrt{y}$ at the point (1,1).
   In what direction does it occur?

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3. Find all the second partial derivatives of

   $f(x,y) = 4x^3y - 7xy^2$

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4. Find the critical points of the function.

   $f(x,y) = 5 + 76xy + 38x^2 + 240y + \frac{y^4}{4}$

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5. Find the local maximum, and minimum value and saddle points of the function.

   $f(x,y) = x^2 - xy + y^2 - 9x + 6y + 10$

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6. Use Lagrange multipliers to find the maximum and the minimum of *f* subject to the given constraint(s).

   $f(x,y) = xyz; \qquad x^2 + y^2 + z^2 = 3$

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7. Find the volume of the given solid.

   Under the paraboloid $z = x^2 + 3y^2$ and above the rectangle $R = [0, 3] \times [1, 5]$.

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8. Evaluate the double integral.

   $$\iint_D x \cos y \, dA$$

   D is bounded by $y = 0$, $y = x^2$ and $x = 2$.

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9. Evaluate the integral by changing to polar coordinates.

   $$\iint_D e^{-x^2 - y^2} \, dA$$

   D is the region bounded by the semicircle $x = \sqrt{16 - y^2}$ and the y-axis.

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10. A cylindrical drill with radius 1 is used to bore a hole through the center of a sphere of radius 5. Find the volume of the ring-shaped solid that remains. Round the answer to the nearest hundredth.

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11. Use spherical coordinates.

    Evaluate $\iiint_B \left(x^2 + y^2 + z^2\right)^2 dV$, where B is the ball with center the origin and radius 4.

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12. Determine whether or not $\mathbf{F}$ is a conservative vector field. If it is, find a function f such that $\mathbf{F} = \nabla f$.

    $\mathbf{F} = \left(8x \cos y - y \cos x\right)\mathbf{i} + \left(-4x^2 \sin y - \sin x\right)\mathbf{j}$.

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13. Find a function f such that $\mathbf{F} = \nabla f$, and use it to evaluate $\int_C \mathbf{F}.\, d\mathbf{r}$ along the given curve C.

$\mathbf{F}(x,y) = x^5y^6\mathbf{i} + y^5x^6\mathbf{j}$

$C: \mathbf{r}(t) = \sqrt{t}\,\mathbf{i} + \left(1 + t^3\right)\mathbf{j},\ 0 \le t \le 1$

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14. Use Green's Theorem to evaluate $\int_C x^2y\,dx - xy^2\,dy$, where C is the circle $x^2 + y^2 = 9$ with counterclockwise orientation.

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15. Find the curl of $x^2z\mathbf{i} + 0\mathbf{j} + xz^2\mathbf{k}$.

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16. Evaluate the surface integral $\iint_S \mathbf{F}.\, d\mathbf{S}$ for the given vector field $\mathbf{F}$ and the oriented surface S. In other words, find the flux of F across S.

$\mathbf{F}(x,y,z) = 4x\mathbf{i} + 4y\mathbf{j} + 4z\mathbf{k}$, S is the sphere $x^2 + y^2 + z^2 = 3$.

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17. Use Stoke's theorem to calculate the surface integral $\iint_S \mathbf{F}.\, d\mathbf{S}$, where

$\mathbf{F}\left(x,y,z\right) = \left(e^x \sin y\right)\mathbf{i} + \left(e^x \cos y - z\right)\mathbf{j} + y\mathbf{k}$, and S is the part of the cone $z^2 = x^2 + y^2$ for which $1 \le z \le 2$.

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18. Calculate the work done by the force field $\mathbf{F}(x,y,z) = \left(x^2 + 10z^2\right)\mathbf{i} + \left(y^2 + 14x^2\right)\mathbf{j} + \left(z^2 + 12y^2\right)\mathbf{k}$ when a particle moves under its influence around the edge of the part of the sphere $x^2 + y^2 + z^2 = 16$ that lies in the first octant, in a counterclockwise direction as viewed from above.

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